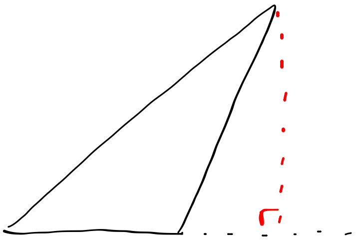
Review

conjecture - basing a prediction or statement on a pattern

inductive reasoning - the thought process of making the conjecture.

Counterexample ~ doesn't fit the conjecture.



$\sqrt{9} = 3$ ✓

$\sqrt{1} = 1$ ✗

p23/14

$10 = 1+2+3+4$

$12 = 3+4+5$

$9 = 4+5$

$94 = 22+23+24+25$

✗ 0+1

1 ?

✗ 0+2 or 1+1

2 ?

← counterexamples

$4 = 2+2$ ✗ or $4 = 1+3$ ✗

$8 = \cancel{3+5}$ or $8 \cancel{=} 4+4$

§1-4 Proving Conjectures: Deductive Reasoning (p27)

*Prove mathematical statements using a logical argument

LEARN ABOUT the Math (p27)

Sum of 5 consecutive integers

median

$1 + 2 + 3 + 4 + 5 = 15 = 5(3)$

median

$(-15) + (-14) + (-13) + (-12) + (-11) = -65 = 5(-13)$

median

$(-3) + (-2) + (-1) + 0 + 1 = -5 = 5(-1)$

Jon's conjecture => the sum of five consecutive integers is five times the median

We could look at many more examples and still not find a counterexample. This does not prove Jon's conjecture.

We need to use deductive reasoning to prove his conjecture mathematically.

Example 1 - Prove that Jon's conjecture is true for all integers

Let x represent any integer

Let S represent the sum of five consecutive integers.

median

$S = x + (x+1) + (x+2) + (x+3) + (x+4)$

$S = 5x + 10$

$S = 5(x+2)$ median

∴ The sum is five times the median

OR

median

$S = (x-\cancel{2}) + (x-\cancel{1}) + x + (x+\cancel{1}) + (x+\cancel{2})$

$S = 5x$

median

Inductive reasoning was used to make the conjecture.

Deductive reasoning was used to prove the conjecture.

APPLY the Math

Example 2 - Prove Steffan's conjecture that the difference between consecutive squares is always an odd number.

Try: $26^2 - 25^2 = 51$ ✓

Prove it:

Let x be any natural number

Let D be the difference between consecutive squares.

$$D = (x+1)^2 - x^2$$

$$x^2 - (x-1)^2$$

$$D = (x+1)(x+1) - x^2$$

$$D = x^2 + x + x + 1 - x^2$$

$$D = \cancel{x^2} + 2x + 1 - \cancel{x^2}$$

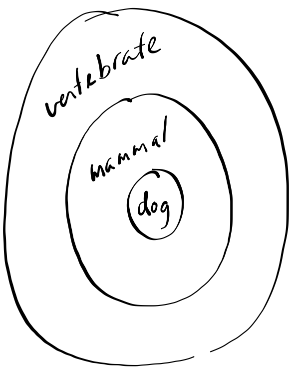
$$D = 2x + 1$$

odd numbers $2x+1$

even number $2x$

The difference is an odd number

Example 3 - What can you deduce about Shaggy?



Shaggy is a dog (given)

Shaggy is a mammal

Shaggy is a vertebrate.

Your Turn -> Inez is building muscle. We cannot deduce that Inez is strong or has improved balance.

BE CAREFUL!

Algebraic Proofs

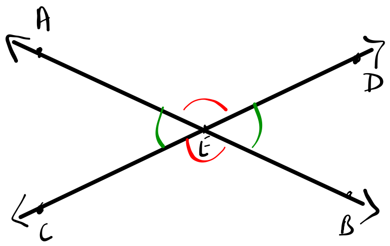
Example 1: $6x + 2(x-1) = 30$

Algebraic Step	Properties
$6x + 2(x-1) = 30$	given
$6x + 2x - 2 = 30$	distributive property.
$8x - 2 = 30$	substitution
$8x - 2 + 2 = 30 + 2$	addition property
$8x = 32$	substitution
$\frac{8x}{8} = \frac{32}{8}$	division property
$x = 4$	substitution

Example 2: Given: $a+b=2c$
$b=c$
Prove: $a=c$

Statement	Jusification
$a+b=2c$	given
$b=c$	given
$a+c=2c$	substitution property
$a+c-c=2c-c$	Subtraction property.
$a=c$	substitution

Example 4 (p29) - Prove that when two straight lines intersect, the vertically opposite angles are equal.



Statement	Justification
$\angle AED + \angle DEB = 180°$	Supplementary angles
$\angle DEB = 180° - \angle AED$	subtraction property.
$\angle AED + \angle AEC = 180°$	Supplementary angles.
$\angle AEC = 180° - \angle AED$	subtraction property
$\angle AEC = \angle DEB$	transitive property.

QED

"quod erat demonstrandum"

"that which was to be demonstrated"

* Transitive property - if two quantities are equal to the same quantity then they are equal to each other

if $a = b$ and $b = c$

then $a = c$

TO DO:

1. CYU (p31) ⇒ all
2. Practising (p31) ⇒ 4-10